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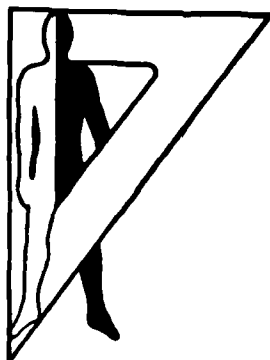
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A COMPUTER SIMULATION MODEL OF OUTDOOR  
RADIANT LIGHTING

Christopher C. Smyth

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<p>A deterministic computer model of outdoor daytime (nighttime) radiant lighting is described for a clear sky and slight ground-level haze. Equations are developed for the radiance at an observer's position, which is emitted from a target and the surrounding background. The equations are a function of the positions of the sun (moon), the target and the observer. The equations include the effects of atmospheric attenuation, the back-ground skylight, and the ground cover. The computer model was developed for use in simulation studies of target visibility.</p>		

(6) A COMPUTER SIMULATION MODEL OF OUTDOOR  
RADIANT LIGHTING

(9) *Fingerprint*

(10) Christopher C./Smyth

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APPROVED:

*John D. Weisz*  
JOHN D. WEISZ

Director

U.S. Army Human Engineering Laboratory

U. S. ARMY HUMAN ENGINEERING LABORATORY  
Aberdeen Proving Ground, Maryland 21005

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## CONTENTS

INTRODUCTION . . . . .	3
DISCUSSION . . . . .	4
Sky Radiance Equations . . . . .	4
Ground Radiance . . . . .	14
Target Radiance . . . . .	16
Background Radiance . . . . .	20
Telescopic Coating Irradiance . . . . .	23
Summary of Radiance Equations . . . . .	24
Target-Background Radiance . . . . .	25
Geographic Coordinates . . . . .	27
CONCLUSION . . . . .	27
REFERENCES . . . . .	28
APPENDIXES	
A. Spectral Data for Daylight, Natural Terrains and Atmospheric Attenuation . . . . .	31
B. Computer Program . . . . .	35
LIST OF NOTATIONS AND SYMBOLS . . . . .	43
FIGURES	
1. Outdoor Lighting Coordinate System . . . . .	5
2. Primary Scattered Light . . . . .	7
3. Multiple Scattered Light . . . . .	8
4. Ground Irradiance . . . . .	15
5. Target Irradiance From the Sky . . . . .	17
6. Target Irradiance From the Ground . . . . .	19
7. Background Irradiance From the Sky . . . . .	21
8. Background Irradiance From the Ground . . . . .	22
9. Apparent Target-Background Radiance . . . . .	26
TABLE	
1. Radiance Equations . . . . .	24

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## A COMPUTER SIMULATION MODEL OF OUTDOOR

### RADIANT LIGHTING

#### INTRODUCTION

The following simplified problem is considered. An observer standing on a flat, infinitely-extending ground surface is sighting with a telescope in the direction of a panel-type target. The ground layer is assumed to be covered with a material of uniform composition having a diffusive reflectance,  $r_g$ , which is spectrally dependent. The target panel is a small vertical surface coated with a diffusively-reflecting material having a spectrally dependent reflectance,  $r_t$ . The panel is standing against a sloped incline as background. The background material has a diffusive, spectrally dependent reflectance,  $r_b$ . The exterior surface of the objective lens piece in the telescope is partially coated with a material having spectrally dependent coefficients,  $t_\lambda$  and  $s_\lambda$ , for light transmittance and scattering, respectively.

The radiant energy reaching the telescope from the target and its background is a function of the following variables:

1. The irradiance from the sky and surrounding ground, which impinges upon the target and the background surfaces.
2. The reflectance values of the target and background surfaces.
3. The radiance from the horizontal sky behind the target and background.
4. The atmospheric extinction (attenuation) coefficient at the ground level.
5. The observer-to-target and observer-to-background distances.

The radiant energy reaching the telescope is attenuated and scattered by the coating on the exterior surface of the objective lens. The distribution of this energy in the telescopic image, and the resulting luminance and chromatic contrasts between the target and background, determine the visibility of the target.

The objective of this report is to establish equations for the apparent radiance at the telescope from the target and background as functions of the sky irradiance and sun angles, and the other physical and geometrical variables. The following sections establish sky radiance equations as a function of sky irradiance and sun angles for a clear sky. The ground radiance, target radiance, background radiance, telescope irradiance and the apparent target and background radiances at the telescope are next derived. Finally, the observer-target coordinate system is related to the geographical coordinate system of the global world.

## DISCUSSION

### Sky Radiance Equations

A review of the literature shows that extensive analysis and measurements have been completed in the field of sky lighting. Chandrasekhar (2) used numerical techniques to develop tables listing the intensity and polarization of a pure air, planar atmosphere. Earlier work assuming vertical light fluxes, as well as natural light, had been developed for pure air (29) and for overcast skies (21). The effects of the earth's curvature and shadow, important during twilight, have also been considered (27). In addition, numerous measurements have been recorded of the daytime, twilight and nighttime skies (12, 16, 17, 18, 19, 20, 25 and 29).

We shall follow the analysis developed by Tousey and Hulburt (29) to derive equations for the sky radiance. They made several simplifying assumptions: (1) the sky light is natural and Stokes' parameters for polarized light may be ignored, (2) the atmosphere has a uniform composition, (3) the atmosphere is bounded top and bottom by infinitely-extending parallel planes, and (4) the diffused light in the atmosphere is described by opposing flux flows in the vertical direction. Chandrasekhar (2) has shown that the assumption of vertical light fluxes do not properly describe the angular distribution of diffused light in planar atmospheres. Furthermore, the assumption of planar atmospheres is not applicable to twilight since the effects of the earth's curvature and shadow upon atmospheric lighting is ignored. Tousey and Hulburt used their analysis to develop brightness and polarization distributions for the daytime sky. Their results did not correctly specify the polarization in all sectors of the sky, but the brightness values were only in slight error (29).

Tousey and Hulburt considered a planar atmosphere of uniform composition and constant thickness,  $z_1$ . The top of the atmosphere is irradiated with sunlight of intensity,  $I_{0\lambda}$ , impinging at a zenith angle,  $\zeta_s$ . The penetrating sunlight is partially scattered by the intervening atmospheric particles in its passage to the ground surface. The scattered light is diffused throughout the atmosphere by further multiple scatterings. The diffused light and sunlight reaching the ground cover is partially reflected back into the atmosphere, thereby generating opposing fluxes of diffused sky light.

The observer is located on the ground at the origin of a three-dimensional cartesian coordinate system (Figure 1). The target is assumed to be near ground level and the observer-to-target direction is along the positive x-axis of the coordinate system. The vertical direction defines the positive z-axis. A sky point is determined by a zenith angle,  $\zeta$ , measured from the vertical z-axis, and the azimuth angle  $\alpha$ . The azimuth angle is measured clockwise about the positive z-axis from the observer-target line. The radiant energy in a differential interval about the wavelength,  $\lambda$ , emitted per unit solid angle from the sky point  $(\zeta, \alpha)$  is described by the function  $N_\lambda(\zeta, \alpha)d\lambda$ . This term is a function of the sun or moon position  $(\zeta_s, \alpha_s)$  in the sky, atmospheric conditions (i.e., ground haze, cloud cover, etc.) and the reflectance value of the ground cover.



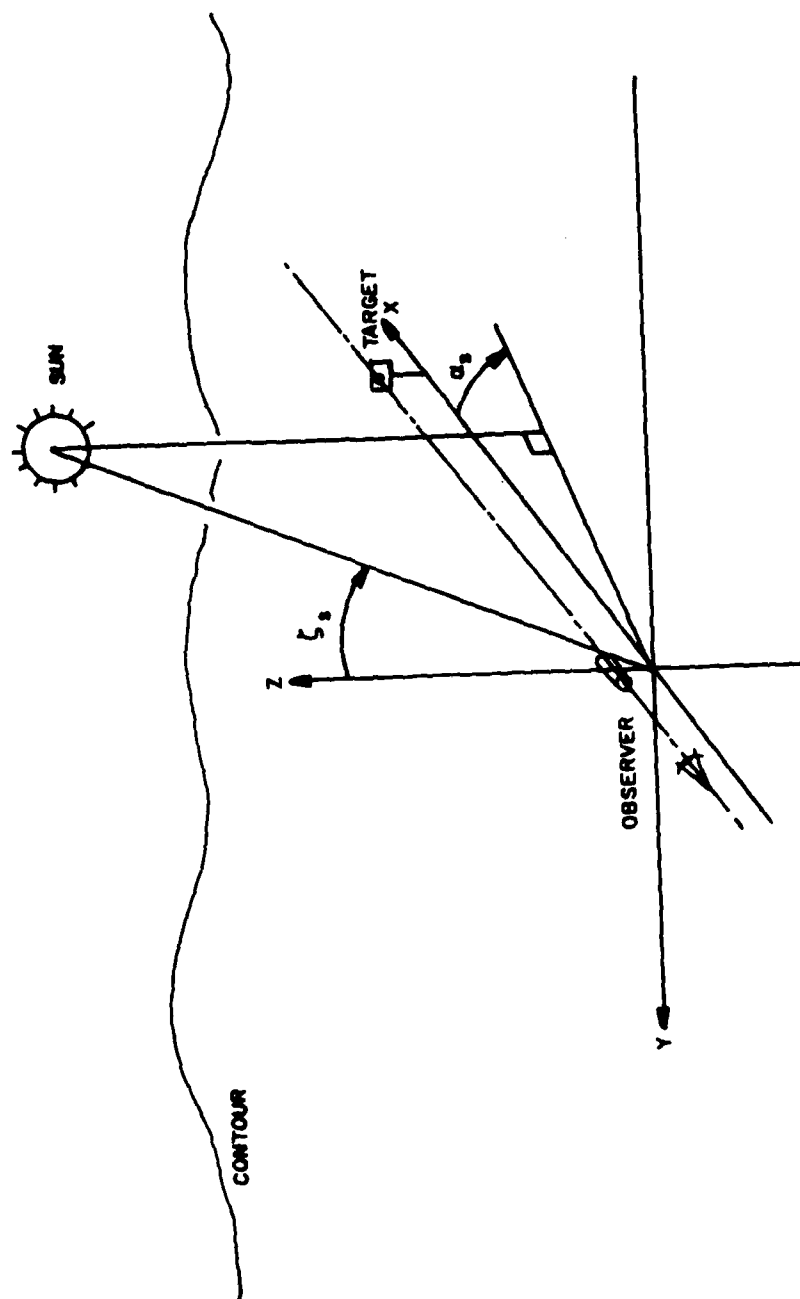


Figure 1. Outdoor lighting coordinate system.

The sky light reaching the observer from the point  $(\zeta, a)$  is the sum of primary scattered light and multiple scattered light. The primary scattered light is sunlight which has been scattered toward the observer by all atmospheric particles along the direction  $(\zeta, a)$ . Let  $I_{O\lambda}$  denote the intensity of a collimated beam of monochromatic sunlight at the top of the atmosphere. The intensity of sunlight which reaches the atmospheric height  $z$  is given by (Figure 2):

$$I_{\lambda} = I_{O\lambda} \exp \left( (\sec \zeta_s) \int_{z_t}^z \beta_{\lambda} d\xi \right),$$

where  $\beta_{\lambda}$  is the atmospheric extinction coefficient and  $\exp$  is the exponential functional (e). The amount of this sunlight scattered toward the observer is this quantity times the atmospheric volume scattering coefficient,  $\sigma_{\phi\lambda}$ . The height  $z$  is on the line  $(\zeta, a)$ , and the scattering angle,  $\phi$ , between this line and the impinging sunlight  $(\zeta_s, a_s)$  is given by:

$$\cos \phi = \cos \zeta_s \cdot \cos \zeta + \sin \zeta_s \cdot \sin \zeta \cdot \cos(a - a_s).$$

This primary scattered light is attenuated by a factor

$$\exp \left( (\sec \zeta) \int_z^0 \beta_{\lambda} d\xi \right),$$

due to multiple scattering in its passage to the observer. The total primary radiance reaching the observer along the direction  $(\zeta, a)$  is given by the integral,

$$N_{\lambda}^P = I_{O\lambda} \int_{z_t}^0 \sigma_{\phi\lambda} \exp \left( (\sec \zeta_s) \int_{z_t}^z \beta_{\lambda} d\xi \right) \exp \left( (\sec \zeta) \int_z^0 \beta_{\lambda} d\xi \right) \sec \zeta dz \quad (1)$$

The multiple scattered light, on the other hand, is diffused light scattered to the observer from the atmosphere (Figure 3). Let  $S_{\lambda}$  denote the sum of the flux per unit area of diffused light flowing vertically in both the upward and downward direction at the atmospheric height,  $z$ . The amount of flux per unit solid angle is  $\beta_{\lambda} S_{\lambda} / 4\pi$ . The quantity reaching the observer along the direction  $(\zeta, a)$  is this amount times the attenuation factor:

$$\exp \left( (\sec \zeta) \int_z^0 \beta_{\lambda} d\xi \right)$$

The total radiance due to multiple scattering which reaches the observer from all volume elements along the direction  $(\zeta, a)$  is given by the integral,

$$N_{\lambda}^m = \frac{1}{4\pi} \int_{z_t}^0 \beta_{\lambda} S_{\lambda} \exp \left( (\sec \zeta) \int_z^0 \beta_{\lambda} d\xi \right) \sec \zeta dz. \quad (2)$$

Tousey and Hulburt make two assumptions: (1) the atmospheric coefficients are constant independent of the vertical  $z$ -coordinate, and (2) the distribution of diffuse radiation may be approximated by upward and downward fluxes of radiation along the  $z$ -axis. Equation (1) for the primary radiation reduces to:

$$N_{\lambda}^P = I_{O\lambda} \cdot \frac{\sigma_{\phi\lambda}}{\beta_{\lambda}} \frac{e^{-\beta_{\lambda} z_t \sec \zeta_s} \cdot e^{-\beta_{\lambda} z_t \sec \zeta}}{1 - \sec \zeta_s \cdot \cos \zeta} \quad (3)$$

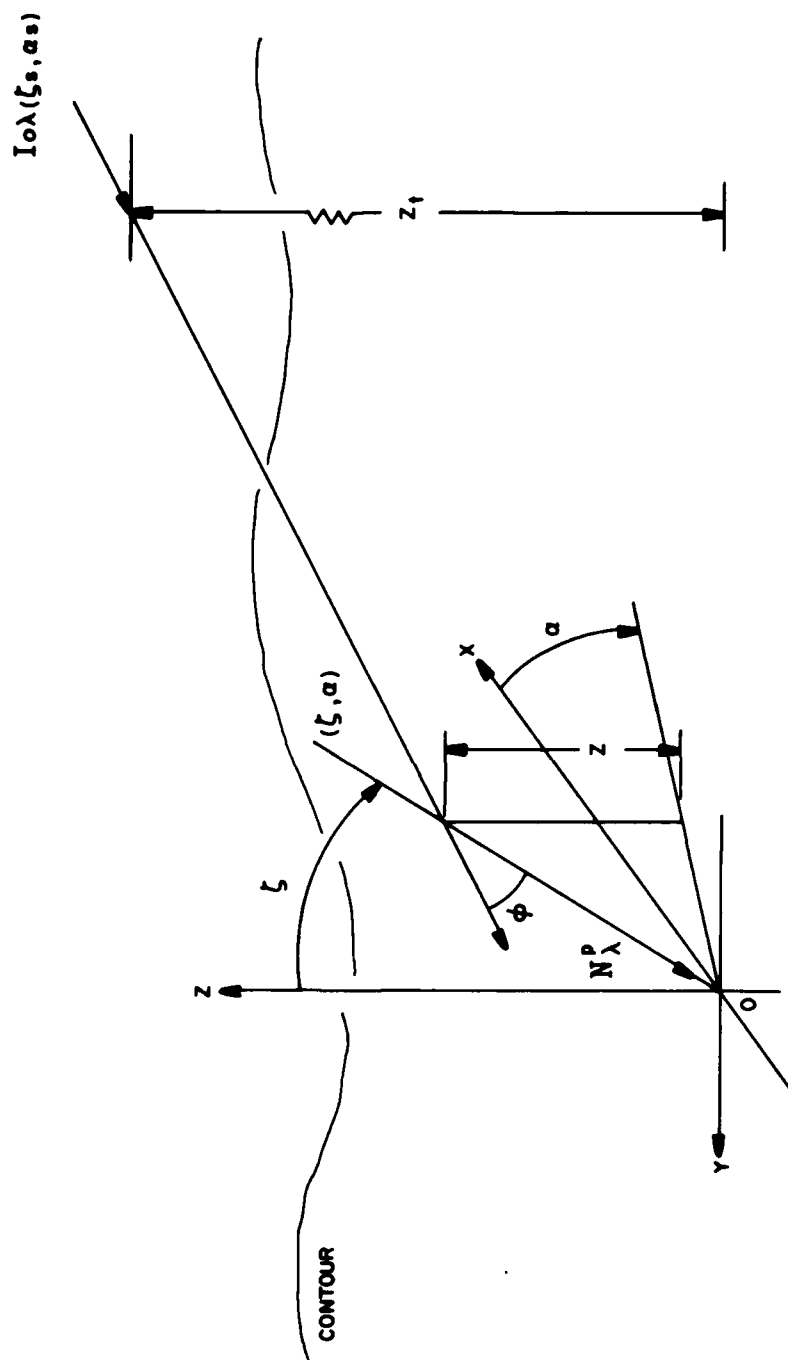


Figure 2. Primary scattered light.

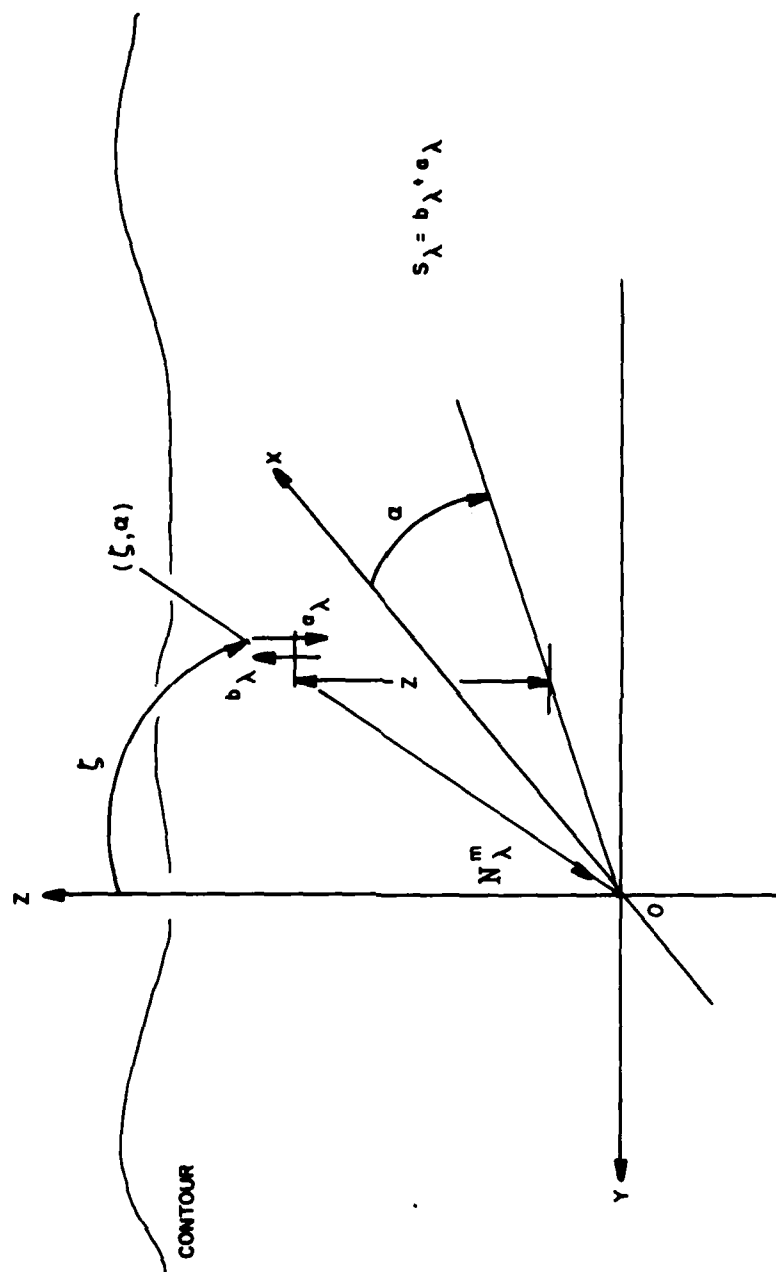


Figure 3. Multiple scattered light.

where the scattering angle,  $\phi$ , is the angular difference between the line  $(\zeta, a)$  and the direction of the incident sunlight  $(\zeta_s, a_s)$ . The equivalent vertical atmosphere thickness,  $Z_t$ , is calculated from the US Standard Atmosphere Data (29).

Tousey and Hulburt solve equation (2) for the multiple-scattered radiance by assuming that the atmosphere is a non-absorbing medium, and considering the boundary conditions on the vertical fluxes of the scattered light at the top and bottom of the atmosphere (5, 10, 29). Equation (2) for the multiple-scattered radiance reduces to:

$$N_{\lambda}^m = \frac{I_{0\lambda}}{4\pi} \frac{\cos \zeta_s}{1+gZ_T} \left\{ CZ_T + \left(a - \frac{c}{\beta_{\lambda}} \cos \zeta\right) (1 - e^{-\beta_{\lambda} Z_T \sec \zeta}) \right. \\ \left. + b \cdot \frac{(e^{-\beta_{\lambda} Z_T \sec \zeta_s} - e^{-\beta_{\lambda} Z_T \sec \zeta})}{(1 - \sec \zeta_s \cos \zeta)} \right\} \quad (4)$$

The equation constants are given by:

$$a = 2(1+gZ_T)k - (k - kT + T)(1 - rg),$$

$$b = (1+gZ_T)(1-2k),$$

$$c = -2g(k - kT + T),$$

$$g = (1 - r_g)(1 - n)\beta_{\lambda},$$

$$k = \eta + (1 - \eta) \cos \zeta_s,$$

$$T = e^{-\beta_{\lambda} Z_T \sec \zeta_s}.$$

Equations (3) and (4) are functions of the volume-scattering coefficient,  $\sigma_{\phi\lambda}$ , the extinction coefficient,  $\beta_{\lambda}$ , and the forward-scattering coefficient,  $\eta$ . These atmospheric coefficients are functions of the type, size, and number of particles in the atmosphere, including air molecules and aerosol particles, as well as water vapor. The volume-scattering coefficient,  $\sigma_{\phi\lambda}$ , determines the proportion of monochromatic light of wavelength,  $\lambda$ , which is scattered by a unit volume of material in a direction at an angle,  $\phi$ , to that of the incident light. The extinction coefficient,  $\beta_{\lambda}$ , is the total proportion of light scattered by a unit volume, and is defined by the angular integration of the volume-scattering coefficient according to:

$$\beta_{\lambda} = 2\pi \int_0^{\pi} \sigma_{\phi\lambda} \sin \phi d\phi.$$

The forward-scattering coefficient,  $\eta$ , is that proportion scattered in the direction of the incident light.

Air molecules scatter light as described by Rayleigh's Law, (22, 29) and the volume-scattering coefficient is given by:

$$\sigma_{\phi\lambda}(\text{air}) = c \cdot \lambda^{-4} \cdot (1+k \cdot \cos^2\phi), \quad (5)$$

where  $k$  accounts for polarization defects and is nearly equal to unity, and  $c$  is a function of particle density. The extinction coefficient is given by:

$$\beta_{\lambda}(\text{air}) = \frac{2\pi}{3} c \cdot \lambda^{-4} (6+2k) \approx \frac{16\pi c \lambda^{-4}}{3}$$

Equation (5) shows that air molecules scatter as much light backwards as forward along the incident light direction, and the forward-scattering coefficient is equal to  $\eta = 0.5$  for a pure air atmosphere.

Now let us consider the case of an atmosphere with pure air and light haze. In general, the atmospheric-extinction coefficient has the form  $\beta_{\lambda} = c(\lambda_0/\lambda)^n$ , where  $\lambda_0$  is the wavelength of green light,  $\lambda_0 = 0.53\mu$ . For light haze,  $c$  ranges from 0.2 to 0.1, and the exponent  $n$  has a corresponding range of 2.0 to 1.3. The ratio of the wavelengths of green light to that of red is 0.82, and that of green to blue is 1.15. The mean value of the atmospheric extinction coefficient is  $\beta \approx 0.02 \text{ km}^{-1}$  (22, 29). The equivalent atmospheric thickness is  $Z_T \approx 8$  kilometers (29). Note that for these values, the exponential function  $\exp(-\beta Z_T \sec\theta)$  may be approximated by  $1 - \beta Z_T \sec\theta$  for values of  $\theta$  equal to or smaller than 70 degrees.<sup>1</sup>

---

<sup>1</sup> Air molecules are much smaller in size than the wavelengths of light waves. Water particles tend to increase in size and number with increasing relative humidity. Assuming spherical particles and little absorption, which is true for water vapor, the scattering coefficient for a given wavelength,  $\lambda$ , changes with particle size,  $a$ , in accordance with Mie's electromagnetic scattering theory (14, 22, 30). As particle size increases, the angle of minimum scattering shifts from  $\phi$  equal to 90 degrees for Rayleigh's Law, backward toward  $\phi$  equal to 120 degrees. Concurrently, the scattering distribution, symmetrical about the plane defined by  $\phi$  equal to 90 degrees, shifts forward toward the incident light direction,  $\phi$  equal to zero. In addition, the dependency of the scattering coefficient upon wavelength changes. Tables listed in Middleton (22) suggest that the scattering coefficient may be approximated by:

$$\sigma_{\phi\lambda}(a/\lambda) = C \lambda^n (1+k \cos^a(f\phi)), \quad (35)$$

where  $k$ ,  $a$ , and  $f$  tend to increase with increasing values of  $a/\lambda$ . The exponent  $n$  increases from  $-4$  for Rayleigh's Law to a small positive number, and then describes a decaying oscillation about the  $n=0$  line for large values of  $a/\lambda$  (22).

In general, the atmospheric-scattering coefficient is the sum of all coefficients for the different particle types. A similar remark applies to

We now consider the application of equations (3) and (4) derived by Tousey and Hulburt (29) to the outdoor lighting problem for an atmosphere with pure air and light haze.

The equations for sky radiance obtained by summing equations (3) and (4) reduce to:

$$N_{\lambda}(\zeta, \alpha) = \frac{I_{0\lambda}}{4\pi} \frac{\beta_{\lambda} Z_T}{1 + g Z_T} \left\{ \frac{3}{4}(1 + \cos^2 \phi) + r_g \cos \zeta_s \right\} \cos \zeta, \quad (6)$$

where  $\zeta, \zeta_s \approx 70^\circ$ . How is this expression related to the sky irradiance,  $H_{\lambda}^s$ , measured on a white diffusive panel placed in a horizontal position? The sky irradiance is given by the equation:

$$H_{\lambda}^s = \int_0^{2\pi} \int_0^{\pi/2} N_{\lambda}(\zeta, \alpha) \cos \zeta \sin \zeta d\zeta d\alpha,$$

and since the contribution from the horizontal sky is insignificant, we use equation (6) to obtain:

$$H_{\lambda}^s = \frac{I_{0\lambda}}{2} \frac{\beta_{\lambda} Z_T}{1 + g Z_T} (1 + r_g \cos \zeta_s). \quad (7)$$

Note that the sky irradiance depends upon the ground reflectance,  $r_g$ , and the sun elevation,  $\zeta_s$ , through the cosine term and also indirectly from the  $I_{0\lambda}$  term.

In general, the sky radiance is expressed in terms of the sky irradiance by summing equations (3) and (4) following reduction for the atmosphere conditions, and substituting equation (7). The result is:

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the atmospheric extinction coefficient. This is reflected in the mean value of the atmospheric extinction coefficient, which increases from  $0.017 \text{ km}^{-1}$  for pure air to 0.1 for light haze, 1 for dense haze and 10 to 100 for fog. The dependency on wavelength shows a corresponding decrease, becoming independent of wavelength in heavy fog (22). The forward-scattering coefficient,  $\eta$ , shows a similar shift in value, ranging from  $\eta = 0.5$  for pure air (29, 15) to  $\eta = 0.8$  for an extremely turbid medium (21).

$$\begin{aligned}
N_{\lambda}(\zeta, a) = & \frac{H_{\lambda}^0 (\beta_{\lambda} Z_T)^{-1} \cos \zeta_s}{2\pi(1+r_g \cos \zeta_s)} \beta_{\lambda} Z_T a \\
& + (1 + \cos \zeta_s + a(1 - \cos \zeta)) (1 - e^{-\beta_{\lambda} Z_T \sec \zeta}) \\
& + (3/4)(1 + \cos^2 \phi) \sec \zeta_s - \cos \zeta_s \left( e^{\frac{-\beta_{\lambda} Z_T \sec \zeta_s - \beta_{\lambda} Z_T \sec \zeta}{1 - \sec \zeta_s \cos \zeta}} \right)
\end{aligned} \tag{8}$$

where

$$a = 1/2(1-r_g)(1+\cos \zeta_s + (1-\cos \zeta_s) \cdot e^{-\beta_{\lambda} Z_T \sec \zeta_s}).$$

The results lead to equation (9) when the exponential functions are replaced by their linear equivalents, valid when both the sun elevation and sky elevation angles are less than 70 degrees.

$$N_{\lambda}(\zeta, a) = \frac{H_{\lambda}^0}{2\pi(1+r_g \cos \zeta_s)} \left\{ 3/4(1 + \cos^2 \phi) + r_g \cos \zeta_s \right\} \sec \zeta. \tag{9}$$

A similar expression may be derived from equation (8) for the horizontal sky radiance,  $N_{\lambda}^h(a) = N_{\lambda}(\pi/2, a)$  at sun angles  $\zeta_s \leq 70^\circ$ , by setting  $\zeta = \pi/2$  and using  $e^{-\beta_{\lambda} Z_T \sec \zeta_s} \simeq 1 - \beta_{\lambda} Z_T \sec \zeta_s$ .

$$\begin{aligned}
N_{\lambda}^h(a) = & \frac{H_{\lambda}^0 (\beta_{\lambda} Z_T)^{-1}}{2\pi(1+r_g \cos \zeta_s)} \left\{ 3/4(1 + \sin^2 \zeta_s \cdot \cos^2(a - a_s)) + r_g \cos \zeta_s \right. \\
& + 1/2 \beta_{\lambda} Z_T \cdot \left\{ 1 - r_g + (1 + r_g) \cos \zeta_s \right. \\
& \left. \left. - (3/2) \sec \zeta_s \cdot (1 + \sin^2 \zeta_s \cdot \cos^2(a - a_s)) \right\} \right\}
\end{aligned} \tag{10}$$



See Footnote 2 for derivation of the sky luminance and illuminance from equations (9) and (10). See Footnote 3 for derivation of sky lighting equations for overcast cloud conditions.

<sup>2</sup>The sky luminance,  $B(\zeta, a)$  is determined by the sky radiance in accordance with,

$$B(\zeta, a) = \int_0^\infty \psi_\lambda \cdot N_\lambda(\zeta, a) d\lambda, \quad (36)$$

where  $\psi_\lambda$  is the luminosity function for the light-adapted eye. Using equation (9), the sky luminance for  $\zeta, \zeta_s \leq 70^\circ$ , is given by:

$$B(\zeta, a) = \frac{E_s}{2\pi(1+r_g \cos \zeta_s)} \left\{ 3/4(1+\cos^2 \phi) + r_g \cos \zeta_s \right\} \sec \zeta \quad (37)$$

where  $E_s$  is the sky luminance and the ground-reflectance values are constant independent of spectra (i.e., snow or black earth). Using equation (10) in equation (35), the horizontal sky luminance,  $B_h(a)$  for  $\zeta_s \leq 70^\circ$  is given by:

$$B_h(a) = \frac{E_s (\beta_\lambda Z_T)^{-1}}{2\pi(1+r_g \cos \zeta_s)} \left\{ 3/4(1+\sin^2 \zeta_s \cdot \cos^2(a-a_s)) + r_g \cos \zeta_s \right. \\ \left. + (1/2)\beta_\lambda Z_T \cdot \left\{ 1-r_g + (1+r_g) \cos \zeta_s \cdot \right. \right. \\ \left. \left. - (3/2) \sec \zeta_s \cdot (1+\sin^2 \zeta_s \cdot \cos^2(a-a_s)) \right\} \right\} \quad (38)$$

<sup>3</sup>We consider for comparison an atmosphere with a heavily overcast sky. The scattering coefficient is dominated by that for the water particles within the cloud layer. These droplets are large in size (20 microns average diameter) and numerous (5 to 50x10<sup>-6</sup> drops per in<sup>3</sup>) (22, 24). The corresponding extinction coefficient is  $\beta_\lambda \approx 10$  to 100 km<sup>-1</sup>, and it is relatively independent of wavelength (22). The scattering process is dominated by the cloud layer, and the effective atmospheric thickness of the turbid medium,  $t$ . Assuming that  $t$ 's magnitude is on the order of one kilometer or larger, then all exponential functions tend to zero. The sky-radiance is determined by equation (4) for the multiple-scattering process, and this equation reduces to:

$$N_\lambda(\zeta, a) = \frac{I_{\lambda 0} \cos \zeta_s}{4\pi(1+gt)} k (1+r_g) \left\{ 1+2\left(\frac{1-r_g}{1+r_g}\right) \cdot (1-\eta) \cos \zeta \right\} \quad (39)$$

Equation (39) is a function of the zenith angle,  $\zeta$ , and ground reflectance,  $r_g$ . The maximum value occurs at the sky-point directly overhead, and the sky-radiance decreases with the increasing values of zenith angle. The equation is independent of azimuth angle,  $a$ , and assumes a constant value independent of zenith angle when a ground surface is highly reflective as for a snow field ( $r_g=1$ ). The sky irradiance is given by:

### Ground Radiance

The ground surface is assumed to be covered with a material of uniform composition having a diffusive reflectance,  $r_g$ , which is spectrally dependent. The radiance,  $W_\lambda$ , emitted in any direction is:

$$W_\lambda^g = \frac{1}{\pi} r_g H_\lambda^s, \quad (11)$$

where  $H_\lambda^s$  is the sky irradiance. The observer-target distance is assumed to be short enough that the sky-radiance distribution remains the same over all ground points in keeping with previous assumptions. Consider an incremental area of sky located along the line,  $(\zeta, a)$  as measured from a coordinate system transposed to a ground surface element (Figure 4). The sky point emits radiance  $N_\lambda(\zeta, a)$  in the direction of the surface element. Its angular area is  $dw = \sin\zeta d\zeta da$ . The light from the sky point is incident at an angle  $\zeta$  to the ground surface normal. The irradiance at the ground element from the sky point is  $dH_\lambda(\zeta, a) = N_\lambda(\zeta, a) \cos\zeta \sin\zeta d\zeta da$ . The total irradiance from the sky is:

$$H_\lambda^s = \int_0^{2\pi} \int_0^{\pi/2} N_\lambda(\zeta, a) \cos\zeta \sin\zeta d\zeta da.$$

$$H_\lambda^s = \frac{I_{\lambda 0} \cos\zeta_s}{4\pi} \frac{1}{1+gt} k (1+r_g) \left\{ 1 + \frac{4}{3} \left( \frac{1-r_g}{1+r_g} \right) \cdot (1-\eta) \right\} \quad (40)$$

and is a function of the zenith angle,  $\zeta_s$ , and ground reflectance,  $r_g$ . Substituting equation (40) into (39) results in:

$$N_\lambda(\zeta, a) = \frac{H_\lambda^s}{1 + (4/3)f} (1 + 2f \cos\zeta), \quad (41)$$

$$\text{where } f = \left( \frac{1-r_g}{1+r_g} \right) (1-\eta)$$

The sky radiance may be expressed in terms of the horizontal sky radiance  $N_\lambda^h = N_\lambda(\pi/2)$ , as follows:

$$N_\lambda(\zeta) = N_\lambda^h (1 + 2f \cos\zeta), \quad (42)$$

where the parameter  $f$  is the same as in equation (41). Assuming that the ground reflectance is independent of spectra, and introducing the luminosity function for the light-adapted eye, the corresponding sky-luminance is given by:

$$B(\zeta) = B_h (1 + 2f \cos\zeta), \quad (43)$$

where  $B_h$  is the horizontal sky-luminance. At least in form, this is in agreement with the experimental measurements by Moon and Spencer (23) of sky luminance on overcast days.

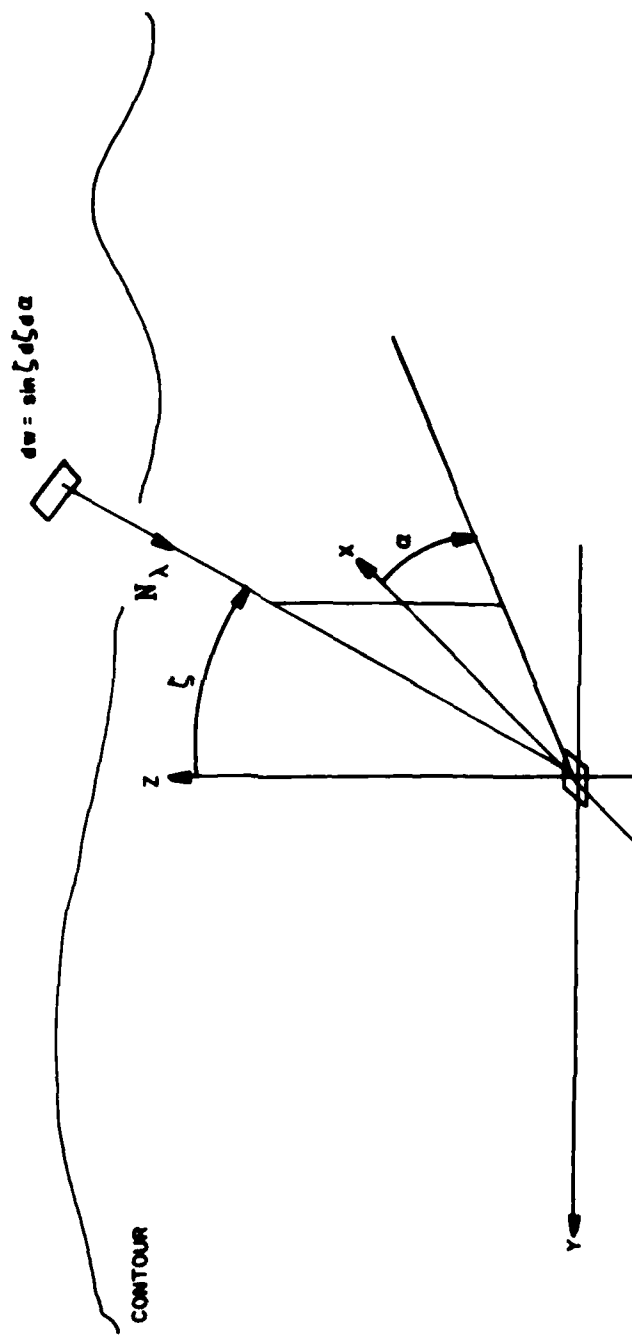


Figure 4. Ground irradiance.

Rather than solve this equation, we note that  $H_{\lambda}^s$  may be measured directly with a spectrometer placed in a horizontal plane.

#### Target Radiance

The target is assumed to be a vertical, flat-panel emplaced at height,  $h_t$ , above the ground level and of such small size that it does not cast a significant shadow upon the ground immediately about it. The target is orientated so that the surface normal is parallel to the x-axis.

The target surface is coated with a diffusively reflecting material having a spectrally dependent reflectance value,  $r_t$ . The radiance emitted in any direction from the target surface is:

$$W_{\lambda}^T = \frac{1}{\pi} r_t H_{\lambda}^T, \quad (12)$$

where  $H_{\lambda}^T$ , the total irradiance at the target surface, is the sum of the irradiance from the sky and that from the ground surface to the target front; i.e.,  $H_{\lambda}^T = H_{\lambda}^{TS} + H_{\lambda}^{TG}$ .

We now derive each of these irradiances in turn and consider their forms for the case of a pure air atmosphere mixed with slight haze. The observer-target distance is assumed to be short enough that the sky radiance distribution remains the same at all ground positions in keeping with previous assumptions. A radiance  $N_{\lambda}(\zeta, \alpha)$  is emitted from the sky point  $(\zeta, \alpha)$  as measured by a coordinate system transposed to the target position (Figure 5). The sky point has a differential solid angular area,  $dw = \sin \zeta d\zeta d\alpha$ . The light from the sky point is at an angle  $\psi$  to the surface normal of the target, where  $\cos \psi = \sin \zeta \cos(\pi - \alpha)$ . The total irradiance at the target surface from the sky is:

$$H_{\lambda}^{TS} = - \int_{\pi/2}^{3\pi/2} \int_0^{\pi/2} N_{\lambda}(\zeta, \alpha) \sin^2 \zeta \cos \alpha d\zeta d\alpha \quad (13)$$

The sky radiance for pure air/slight haze atmosphere is described by equation (8). However, this equation contains exponential functions which produce singularities following integration. For this reason, we approximate equation (8) by equation (9) for sun and sky zenith angles less than 70 degrees; i.e.,  $\zeta_s, \zeta < 70^\circ$ , and by equation (10) for sky angles near the horizon; i.e.,  $70^\circ < \zeta < 90^\circ$ . The target is a vertical surface and as we shall see, only the horizontal sky provides a significant contribution to the sky irradiance. We separate equation (13) for the sky irradiance into the sum of two components: (1) an integration of equation (9) over the range  $0 < \zeta < 70^\circ$ , and (2) an integration of equation (10) over the range  $70^\circ < \zeta < 90^\circ$ .

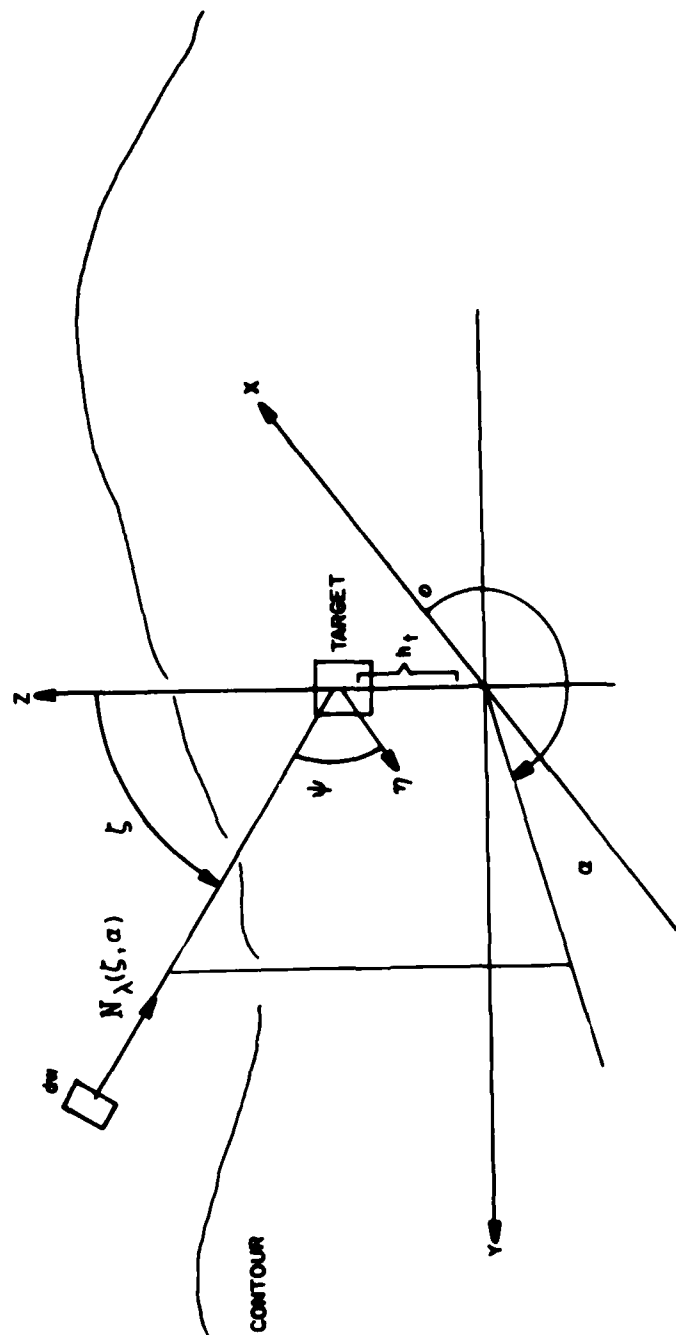


Figure 5. Target irradiance from the sky.

The results of the integration of the first component are:

$$H_{\lambda}^{TS}(1) = \frac{H_{\lambda}^0}{2\pi(1+r_g \cos \zeta_g)} \left\{ 0.135 - (0.372 r_g \cos \zeta_g + 0.79 \cos \zeta_g \sin \zeta_g \cos \zeta_g + 0.645 \sin^2 \zeta_g + 0.231 \sin^2 \zeta_g \cos^2 a_g) \right\} \quad (14)$$

while those for the second component, using  $\beta_{\lambda} z_T \approx 0.16$ , are:

$$H_{\lambda}^{TS}(2) = \frac{H_{\lambda}^0}{2\pi(1+r_g \cos \zeta_g)} \left\{ 1.59 \left\{ 3 + 4 r_g \cos \zeta_g + \sin^2 \zeta_g (1 + \cos^2 a_g) \right\} - 0.255 \left\{ 2(1 - r_g + (1 + r_g) \cos \zeta_g - 1.5 \sec \zeta_g) - \sec \zeta_g \sin^2 \zeta_g (1 + \cos^2 a_g) \right\} \right\} \quad (15)$$

A comparison of the two components shows that the sky irradiance is dominated by equation (15) for the horizontal sky.

We now consider the irradiance received by the target from the ground (Figure 6). The radiance emitted from an incremental element of ground area located at a radius  $\rho$  and angle  $a$  measured from the target's front, is  $W_{\lambda}^0(\rho, a)$  given in equation (11). The straight line distance between the ground element and target is  $l = (\rho^2 + h_1^2)^{1/2}$ , and the ground radiance is attenuated in its passage through the atmosphere by an exponential function of the spectral dependent, atmospheric extinction coefficient,  $\sigma_{\lambda}$ . Furthermore, a certain portion of the horizontal sky light,  $N_{\lambda}(\pi/2, a)$ , is added to the ground radiance. The net radiance at the target from the direction at the ground element is given by:

$$W_{\lambda}^{GT}(\rho, a) = W_{\lambda}^0(\rho, a) e^{-\sigma_{\lambda} l} + N_{\lambda}(\pi/2, a) (1 - e^{-\sigma_{\lambda} l}). \quad (16)$$

The angular area subtended by the ground element is  $d\omega = ds \cos \theta / l^2$ , where  $ds = \rho d\rho da$  and the angle  $\theta$  is measured from the vertical  $z$ -axis to the straight line along the distance,  $l$ ; i.e.,  $\cos \theta = h/l$ . The light from the ground element is at an angle  $\psi$  to the surface normal of the target, where  $\cos \psi = \sin \theta \cos(\pi - a)$ . The total irradiance at the target from the ground is:

$$H_{\lambda}^{TG} = -h_1 \int_{\pi/2}^{\pi/2} \int_{h_1}^{\infty} \left\{ \frac{H_{\lambda}^0 r_g}{\pi} - N_{\lambda}^h(a) \right\} e^{-\sigma_{\lambda} l} + N_{\lambda}^h(a) \left\{ \frac{l^2 - h_1^2}{l^3} \right\}^{1/2} \cos a d\theta da. \quad (17)$$

Equation (17) contains exponential functions which produce singularities when integrated. However, the values of the extinction coefficient,  $\sigma_{\lambda}$ , are small in a pure air atmosphere, and the exponential function decreases at a slower rate with increasing distance, than does the inverse square function,  $1/l^2$ . In other words, the contribution of distinct ground elements becomes insignificant with increasing distance, before appreciable

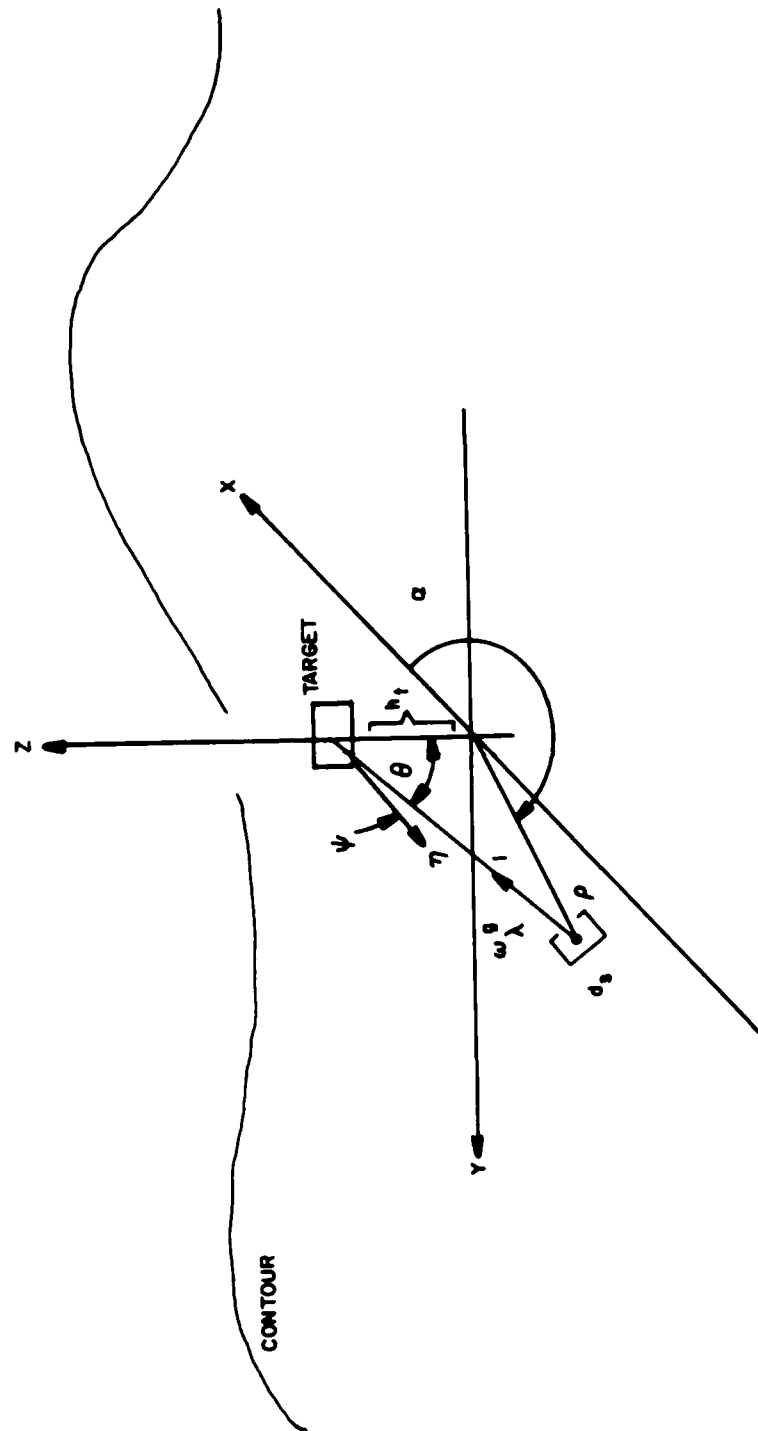


Figure 6. Target irradiance from the ground.

atmospheric attenuation can occur. The horizontal sky radiance terms cancel in the equation, and the resulting integration of the ground radiance is:

$$H_{\lambda}^{Tg} = \frac{1}{2} H_{\lambda}^s \cdot r_g. \quad (18)$$

### Background Radiance

The background visually surrounding the target has a diffusive, spectrally dependent reflectance,  $r_b$ . It slopes upward away from the observer with the surface normal at an angle  $\frac{\pi}{2} - \gamma$  to the vertical  $z$ -axis (Figure 7). The background incline is slight ( $\gamma$  nearly equal to  $\pi/2$ ), and little shadow is cast upon the ground to its immediate front. It does not provide significant radiance to ground elements. The observer-target line intercepts the background surface at a height,  $h_b$ , above ground level. The radiance emitted from the background in the direction of the observer is:

$$W_{\lambda}^b = \frac{H_{\lambda}^b r_b}{\pi}, \quad (19)$$

where the irradiance on the background,  $H_{\lambda}^b$ , is the sum of the irradiance from the sky and that from the ground; i.e.,  $H_{\lambda}^b = H_{\lambda}^{bs} + H_{\lambda}^{bg}$ . We again derive each of these irradiances in turn and consider their forms for the case of a pure air/slight haze atmosphere.

The radiance,  $N_{\lambda}(\zeta, a)$ , is at an angle  $\psi_s$  to the surface normal of the background, where  $\cos \psi_s = \cos \zeta \sin \gamma - \sin \zeta \cos a \cos \gamma$ . The irradiance from a differential element of angular area located at the sky point is  $dH_{\lambda}^{bs}(\zeta, a) = N_{\lambda}(\zeta, a) \sin \zeta \cos \psi_s d\zeta da$ . The total irradiance at the background surface from the sky is:

$$H_{\lambda}^{bs} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^{\frac{\pi}{2}} dH_{\lambda}^{bs}(\zeta, a) + \int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} \int_0^{\cos^{-1}(\cos \gamma \cos a)} dH_{\lambda}^{bs}(\zeta, a) \quad (20)$$

However, equation (20) was derived assuming that angle  $\gamma$  is nearly equal to  $\pi/2$ ; that is, that the surface incline is slight. For this reason, we approximate the sky irradiance at the sloped background by that at a horizontal surface; i.e.,

$$H_{\lambda}^{bs} = H_{\lambda}^s. \quad (21)$$

The radiance from an incremental ground element (Figure 8) on a radius  $\rho$  and angle  $a$ , measured from the surface front, is  $W_{\lambda}^{bg}(\rho, a)$  given in equation (11). The radiance is at an angle  $\psi_g$  to the surface normal, where  $\cos \psi_g = \sin \gamma \cos \theta - \cos \gamma \sin \theta \cos a$ , and the angle  $\theta$  is between the vertical  $z$ -axis and the straight line distance,  $\ell = (\rho^2 + h_b^2)^{1/2}$ . The total ground irradiance is:

$$H_{\lambda}^{bg} = h_b \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^{\infty} \left\{ \left( \frac{1}{\pi} H_{\lambda}^s r_g - N_{\lambda}^h(a) \right) e^{-\sigma_{\lambda} \ell} + N_{\lambda}^h(a) \right\} \cdot (h_b \sin \gamma - (\ell^2 - h_b^2)^{1/2}) \cdot \cos \gamma \cos a \frac{d\ell}{\ell^3} da. \quad (22)$$



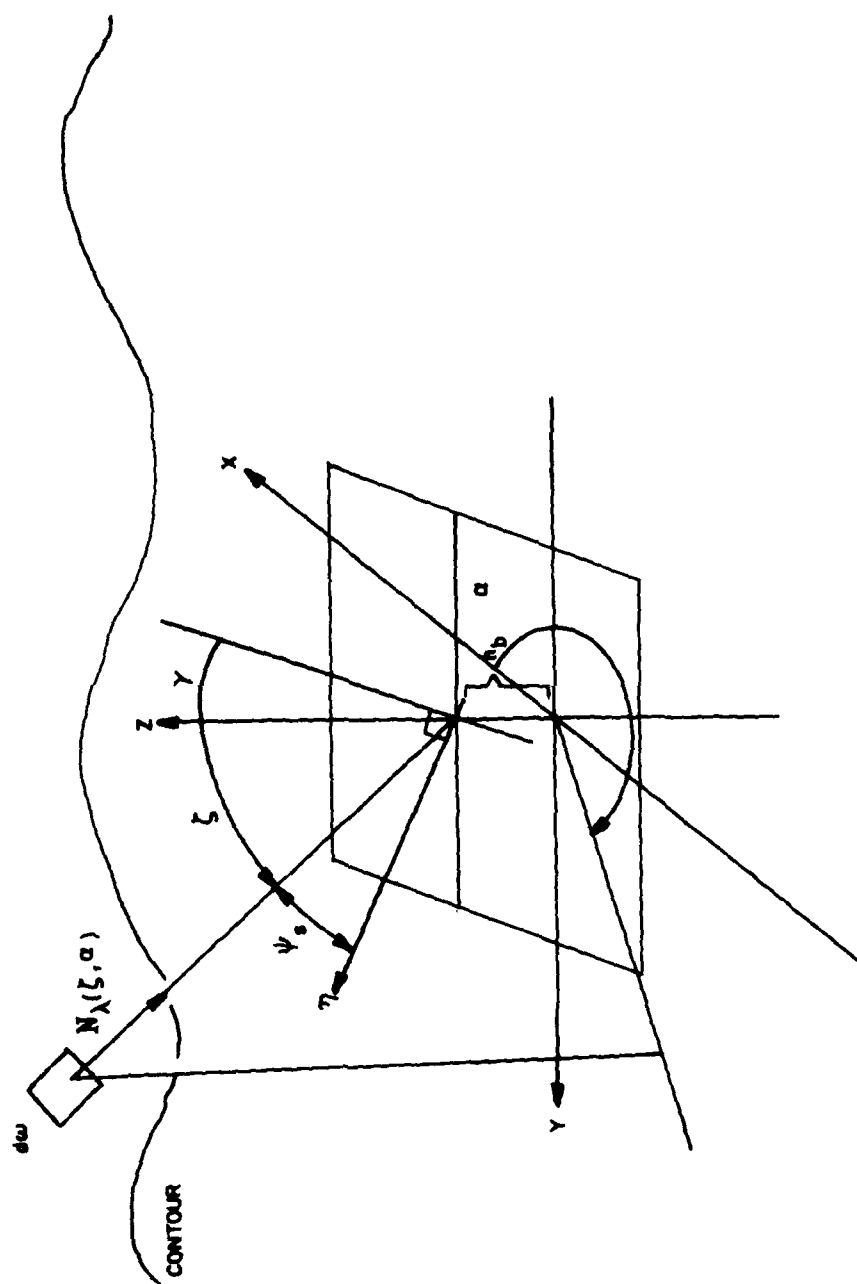


Figure 7. Background irradiance from the sky.

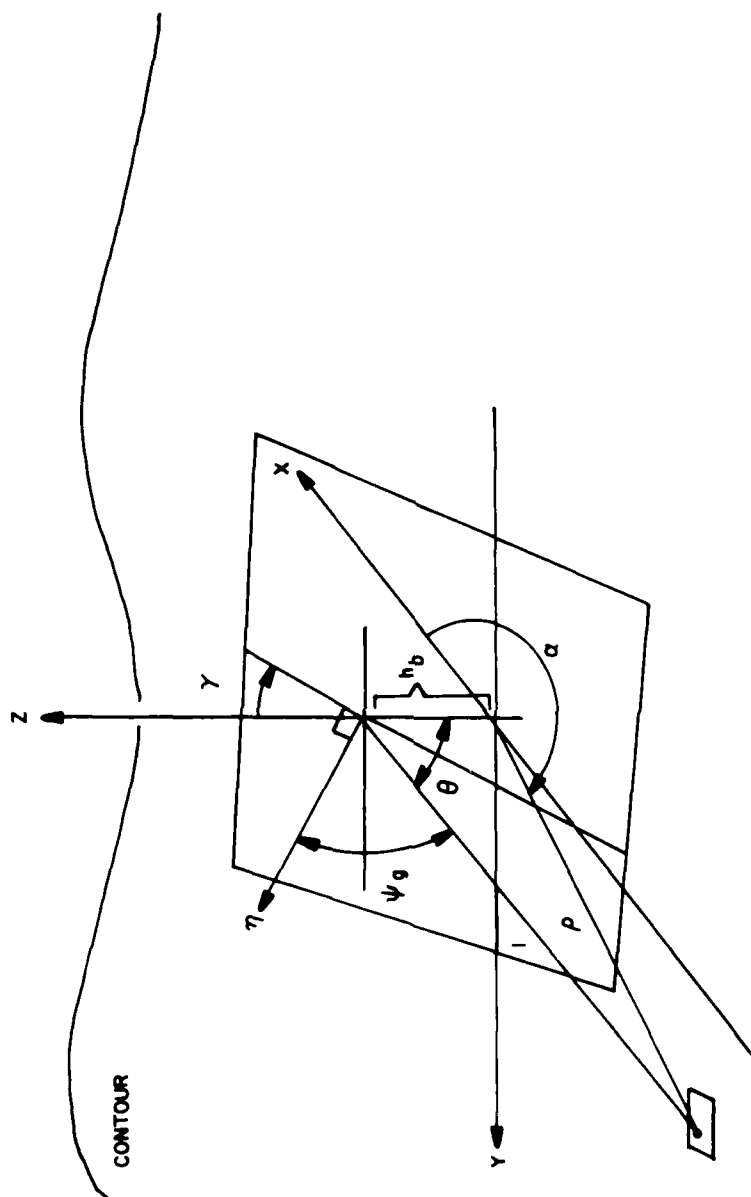


Figure 8. Background irradiance from the ground.

The background surface is assumed to parallel the y-axis; that is, the line of steepest ascent is in the x-z plane. Note that the lower limit of integration of the variable  $\ell$  varies with the angle,  $\alpha$ .

We assume in equation (22) that: (1) the exponential functions may be ignored due to moderate extinction in a pure air atmosphere, and (2) the lower limit of the distance variable,  $\ell$ , is  $h_b \tan \gamma \sec \alpha$ , since the angle  $\gamma$  is nearly equal to  $\pi/2$  and the term  $\tan^2 \gamma \sec^2 \alpha$  is much larger than unity. Equation (22) reduces to:

$$H_{\lambda}^{bg} = \frac{1}{2} H_{\lambda}^s r_g \cos \gamma \cot \gamma. \quad (23)$$

Note that  $H_{\lambda}^{bg} = 0$  when  $\gamma = \pi/2$  as would be expected for a horizontal surface, but that  $H_{\lambda}^{bg}$  approaches infinity as the angle  $\gamma$  approaches zero because of the second assumption above. The total irradiance at the background surface is the sum of equations (22) and (23).

$$H_{\lambda}^b = H_{\lambda}^s (1 + \frac{1}{2} r_g \cos \gamma \cot \gamma). \quad (24)$$

#### Telescopic Coating Irradiance

The exterior surface of the objective lens of the observer's telescope is partially coated with a material having spectral dependent light transmittance and diffusive scattering coefficients. The amount of light scattered by the coating is a function of the irradiance at the exterior surface of the telescope. The determination of this irradiance is equivalent in formulation to that for the target, except that the telescopic front faces away from the observer, instead of toward him. The irradiance is the sum of that from the sky and that from the ground; i.e.,  $H_{\lambda}^c = H_{\lambda}^{cs} + H_{\lambda}^{cg}$ . The irradiance from the sky is:

$$H_{\lambda}^{cs} = \int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} N_{\lambda}(\zeta, \alpha) \sin^2 \zeta \cdot \cos \alpha d\zeta d\alpha, \quad (25)$$

while that from the ground is:

$$H_{\lambda}^{cg} = h_c \int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} \int_{h_c}^{\infty} \left\{ \left( \frac{H_{\lambda}^s r_g}{\pi} - N_{\lambda}^h(\alpha) \right) e^{-\sigma_{\lambda} \ell} + N_{\lambda}^h(\alpha) \right\} \left( \frac{\ell^2 - h_c^2}{\ell^3} \right)^{1/2} \cos \alpha d\ell d\alpha, \quad (26)$$

where  $h_c$  denotes the scope height above ground level.

Equation (25) for the sky irradiance is separable into the sum of two components. The first component is the integration of equation (9) between the limits of  $0^\circ$  and  $70^\circ$  for the zenith angle, while the second component is that of equation (10) between  $70^\circ$  and  $90^\circ$ . The first component is:

$$H_{\lambda}^{CS}(1) = \frac{H_{\lambda}^s}{2\pi(1+r_g \cos \zeta_g)} \left\{ 0.135 - (0.37r_g \cos \zeta_g - 0.79 \cos \zeta_g \cdot \sin \zeta_g \cos a_g + 0.645 \sin^2 \zeta_g + 0.231 \sin^2 \zeta_g \cos^2 a_g) \right\}, \quad (27)$$

and using  $\beta_{\lambda} Z_T \approx 0.16$ , the second component is:

$$H_{\lambda}^{CS}(2) = \frac{H_{\lambda}^s}{2\pi(1+r_g \cos \zeta_g)} \left\{ 1.59(3+4r_g \cos \zeta_g + 2 \sin^2 \zeta_g (1+\cos^2 a_g)) + 0.255(2(1-r_g+(1+r_g) \cos \zeta_g - 3/2 \sec \zeta_g) - \sec \zeta_g \cdot \sin^2 \zeta_g (1+\cos^2 a_g)) \right\}. \quad (28)$$

Equation (27) for the telescopic surface is identical to equation (14) for the target since they are the irradiance from the zenith sky. Equations (15) and (28) give the irradiances from the corresponding horizontal skies, and are identical in form except for the sign of the second bracketed term.

The irradiance from the ground is given by equation (26) assuming that the exponential functions may be ignored in a pure air/slight haze atmosphere. The result is:

$$H_{\lambda}^{CG} = \frac{1}{2} H_{\lambda}^s r_g. \quad (29)$$

Equation (29) is identical to equation (18) for the ground irradiance at the target, as would be expected.

#### Summary of Radiance Equations

The irradiance and radiance equations derived for a pure air/slight haze atmosphere are listed in Table 1 as a function of the emitting surface.

TABLE 1  
Radiance Equations

Surface	Irradiance	Radiance
Ground	- - -	(11)
Target	(14)+(15)+(18)	(12)
Background	(24)	(19)
Telescope	(27)+(28)+(29)	--

### Target-Background Radiance

The radiance emitted from target and background must be adjusted for atmospheric attenuation and addition of air light, to determine those values which reach the observer (22, 3, 4, 6, 7, 8, 9, 11, and 13) (Figure 9). The radiance at the observer from the target and the intervening atmosphere is:

$$W_{\lambda}^{T'} = W_{\lambda}^T e^{-\sigma_{\lambda} \ell_T} + N_{\lambda}^h(0) \cdot (1 - e^{-\sigma_{\lambda} \ell_T}), \quad (30)$$

where the radiance emitted from the target,  $W_{\lambda}^T$ , is given by equation (12), and the horizontal sky radiance,  $N_{\lambda}^h(0)$ , the observational direction is given by equation (10) with the azimuth angle,  $\alpha = 0$ . The straight line distance,  $\ell_T$ , between the observer and target is in the same units as the inverse of the atmospheric extinction coefficient,  $\sigma_{\lambda}$ , along the path of observation.

The radiance at the observer from the background is:

$$W_{\lambda}^{b'} = W_{\lambda}^b \cdot e^{-\sigma_{\lambda} \ell_b} + N_{\lambda}^h(0) \cdot (1 - e^{-\sigma_{\lambda} \ell_b}), \quad (31)$$

where the background radiance,  $W_{\lambda}^b$ , is given by equation (19) and  $\ell_b$  is the straight line distance between the observer and the background.

The observer-background distance ( $\ell_b$ ) is a function of the observer-target distance ( $\ell_T$ ) and the terrain. The straight line connecting the observer's telescope and the target passes through the background position. By congruent triangles,

$$\ell_b = \ell_T \frac{(h_b - h_c)}{(h_T - h_c)}, \quad (32)$$

where  $h_c$ ,  $h_T$ , and  $h_b$  are the vertical heights of the telescope, target and background positions above ground level. The straight line distances are given by:

$$\begin{aligned} \ell_T &= (X_T^2 + (h_T - h_c)^2)^{1/2}, \\ \ell_b &= (X_b^2 + (h_b - h_c)^2)^{1/2}, \end{aligned} \quad (33)$$

where  $X_T$  and  $X_b$  are the horizontal observer-target and observer-background distances. The directional cosines of the connecting line are  $a_x = X_T/\ell_T$  &  $a_z = (h_T - h_c)/\ell_T$ . The apparent angular area of the target subtended at the observer's position is,

$$\omega_T' = A_T a_x / \ell_T^2, \quad (34)$$

where  $A_T$  is the target-area.

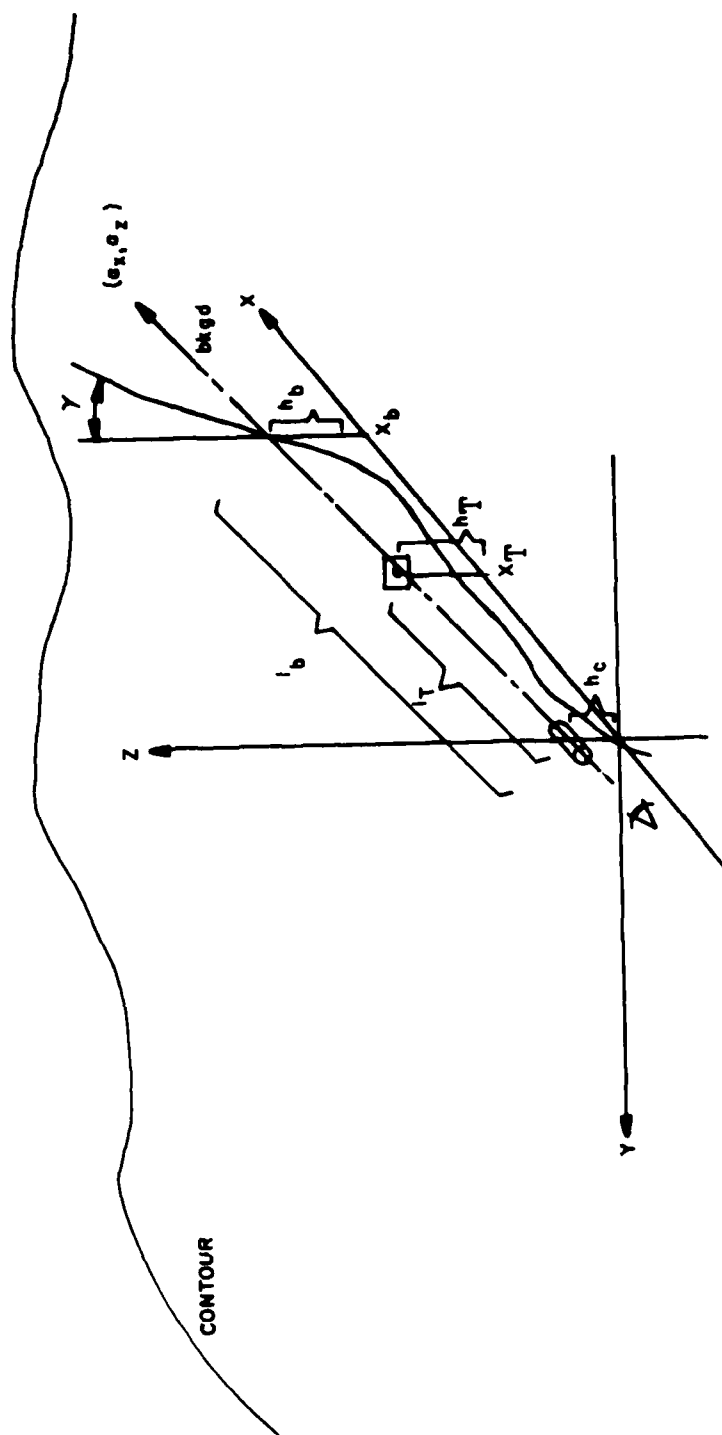


Figure 9. Apparent target-background radiance.

## Geographic Coordinates

The observer's coordinate system specified by the observer-target line may be converted to the geographic coordinates of the global world given by (a) geographical latitude of the observer's position, (b) bearing of the observer-target line from true North, and (c) declination, meridian, and azimuth of the sun. Burge (1) lists equations converting the zenith and azimuth angles of the sun as measured in the observer's coordinate system to appropriate coordinates in the geographic system.

The zenith angle of the sun,  $\zeta_s$ , is converted by  $\cos \zeta_s = \sin \mu \sin \delta + \cos \mu \cos \delta \cos t$ , where the angle  $\mu$  is the geographical latitude of the observer's position. The angle,  $\delta$ , is the declination of the sun, that is, the angular distance of the sun above or below the equator varying with the season. The angle,  $t$ , is the meridian angle measured on the equator from the North Point to the circle passing through the sun. The meridian angle may be computed from the local apparent time  $t_s$ , by  $t = t_s - 12$  hours.

The sun's azimuth angle,  $\alpha_s$ , is converted by  $\alpha_s = \alpha_o - \alpha_N$ , where  $\alpha_o$  is the azimuth of the sun measured from true North as given by  $\sin \alpha_o = (\sin \zeta_s \cos \delta) / \sin \zeta_s$ . The angle  $\alpha_N$  is the bearing of the observer-target line measured from true North.

## CONCLUSION

Equations are derived for the sky radiance of a clear sky with slight haze as a function of the sky irradiance, the sun angles and the spectro-reflectance of the ground cover. These expressions are used to derive equations for the radiances emitted from a panel target and its background as a function of their spectro-reflectance values. The apparent radiances reaching the telescope from the target and background are derived by considering the intervening atmosphere and the distances of the target and background from the observer. The irradiance arriving at the telescope-front from the surrounding sky and ground are derived to complete the lighting scenario. Finally, the observer-target coordinate system is related to the geographic coordinate system of the global world.

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APPENDIX A

SPECTRAL DATA FOR DAYLIGHT, NATURAL TERRAINS AND  
ATMOSPHERIC ATTENUATION

## Spectral Data for Daylight, Natural Terrains and Atmospheric Attenuation

The spectral data for daylight irradiance, spectrereflectances of natural terrain features and the atmospheric attenuation coefficients are briefly reviewed in this section.

### DAYLIGHT IRRADIANCE SPECTRUM

The relative spectral power from the total sky and sun irradiating a horizontal surface changes only slightly over the visual spectrum during daylight. This is true for the sun at altitudes anywhere between two hours after sunrise and two hours before sunset, and the sky anywhere from totally covered with clouds (overcast sky) to totally free of clouds (blue sky). The ICE standard illuminant D65 is based on numerous spectroradiometric measurements of daylight at different locations throughout the world, and best represents natural daylight (Judd and Wyszecki, 1975 [3]).

### SPECTRORREFLECTANCES OF NATURAL TERRAIN FEATURES

The spectral reflectance of natural terrains has been extensively investigated by Krinov (1953) (4). As reported in Pendorf (1954) (5), Krinov took most of his spectral measurements on clear days around noontime. In the case of horizontal terrains (i.e., snow fields, grass, earth), the spectrograph was placed at ground level on a tripod and directed downward at an angle of 45 degrees and an azimuth of 90 degrees from the sun. For vertical terrains such as trees, the spectrograph was directed horizontally and at an azimuth of between 135 degrees to 225 degrees from the sun.

The spectral curves for snow fields are relatively flat. The values for freshly fallen snow drop gradually toward the red portion of the spectrum. The curve for snow covered with a film of ice is flat, independent of wavelength.

The spectral curve for vegetative formations has a weak maximum in the yellow-green region, and an upward slope in the red portion. The curve for coniferous forests in the winter period is relatively flat with a slight rise in the near-infrared region. The curve for coniferous forests in the summer period, dry meadows, and grass (excluding lush grass), is higher in value and shows a similar rise in the near infrared. The curve for deciduous forests in the summer period and all lush grass has a weak maximum in the yellow-green region, and a strong rise in the infrared region. Ripe field crops and forests in autumn show an upward slope in the entire green-to-red portion, and a steep slope in the near infrared portion.

Measurements from the ground and from the air are in agreement for bare ground areas, such as soil and shallow or dense vegetative foundations. However, measurements of forests from the air are only one-half to

one-quarter of those for a ground observer due to the large shadow areas between individual trees. For such vertical vegetative formations, the ground spectral reflectance values used to calculate sky-radiance should be those values measured from the air.

#### ATMOSPHERIC ATTENUATION COEFFICIENTS

The spectrum variations of the atmospheric attenuation coefficient used in this study are a simplification of actual data (Barnes, 1968 [1]), since the line structures of atmospheric absorption due to water vapor, carbon dioxide, etc., have been ignored.

#### REFERENCES

1. Barnes, F.A. Electro-optics handbook. Radio Corporation of America, SCN-102-67, Burlington, MA, 1968.
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3. Judd, D.B., & Wyszecki, G. Color in Business, Science and Industry. New York: John Wiley & Sons, 1975.
4. Krinov, E.L. Spectral reflectance properties of natural formations (in Russian, 1947). National Research Council of Canada Technical Translation TT-439, Ottawa, 1953.
5. Pendorf, R. Luminous and spectral reflectance as well as colors of natural objects (albedo and color of terrain features). Air Force Cambridge Research Center, TR-56-203, February 1956, (AD 93766).

**APPENDIX B**

**COMPUTER PROGRAM**

### Computer Program

A computer program was written in Fortran IV language to compute the apparent radiances from the target and the background. The user must specify the angular position of the sun (moon) relative to the observer and target, the position of the target, and the spectrophlectance values of the ground cover, target, and viewed background.

The program first calculates the sky illuminance (foot-candles), and the luminance (foot-lamberts) from the viewed background at the surface, the luminous reflectance value of the background, and the horizon luminance. These values are printed out for reference. The program then computes the apparent radiances from the background and target.

The computer program is attached below, and a list of subroutines follows:

1. TEST - Main program sets initial conditions and calls for computation of target and background radiances.

2. SUNA - Subroutine computes constants of irradiance and radiance equations as a function of the sun's elevation and azimuth, and the background slope angle. Called by TEST.

3. LUMIN - Subroutine computes sky illumination (foot-candles), the viewed background luminance (foot-lamberts), the luminous reflectance of the background and the horizon luminance. These values are computed from the constants of SUNA and the spectrophlectances of the ground cover and background. Called by TEST.

4. RADT - Subroutine computes the radiances emitted by the target and background at the specified range. Called by TEST.

5. SKYI - Data file of sky irradiance spectrum. Called by LUMIN and RADT.

6. GRDS - Data file of ground cover spectrophlectance spectrum. Called by LUMIN and RADT.

7. TGTS - Data file of target spectrophlectance spectrum. Called by RADT.

8. BKGD - Data file of viewed background spectrophlectance spectrum. Called by LUMIN and RADT.

9. ATTC - Data file of horizontal atmospheric attenuation coefficients for standard clear atmosphere, visibility 23.5 kilometers. Called by RADT.

```

SMYTH,STHFZ,T1000.
ACCOUNT,HE***.
FTN(SL,R,T)
MAP(PART)
LGD.
    PROGRAM TEST(OUTPUT,TAPE3=OUTPUT)
    COMMON/BKGD/ANGB
    COMMON/PADD/XT(79),XB(79),SC(79)
C TARGET DIAMETER AND RANGE IN METERS
    DATA DTO/3./
    DATA R/2000./
C VIEWED BACKGROUND 60-DEGREES FROM GROUND NORMAL (10-DEGREE SLOPE)
    DATA ANGB/1.39626/
C SUN DIRECTLY OVERHEAD
    DATA EL,AZ/2*0./
    WRITE(3,990)
    990 FORMAT(1H,6X,'VISIBILITY COMPUTATIONS')
C SKY LIGHTING CONDITIONS
    CALL SUNA(EL,AZ,ANGB)
    CALL LUMIN(ES,BL,BR,BHS)
    WRITE(3,991)ES,BL,BR,BHS
    991 FORMAT(1H,6X,'SKY ILL(FC)=' ,F10.4,' BKGD LUM(FL)=' ,F10.4,' BKGD R
    CELECTANCE=' ,F10.4,' HORIZON (FL)=' ,F10.4)
C SET UP TARGET AND BACKGROUND RADIANCE FOR SPECIFIED RANGE
    10 RXB=R+DTO*TAN(ANGB)/2.
    WRITE(3,994)R,RXB
    994 FORMAT(2X,'TARGET-RANGE',2X,F10.4,2X,'BACKGROUND RANGE',2X,F10.4)
    CALL RADT(R,RXB)
C WRITE OUT TARGET APPARENT RADIANCE
    WRITE(3,1000)(XT(I),I=1,79)
    1000 FORMAT(2X,10(F10.4,1X))
C WRITE OUT BACKGROUND RADIANCE
    WRITE(3,1000)(XB(I),I=1,79)
C WRITE OUT IRRADIANCE AT SCOPE FRONT
    WRITE(3,1000)(SC(I),I=1,79)
    STOP
    END

```



```

SUBROUTINE SUNA(EL,AZ,BN)
C COMPUTES CONSTANTS OF SKY RADIANCE EQUATIONS
COMMON/CONST/A,AB,BT,CT,DT,ET,BC,CC,DC,EC,BH,CH,DH,EH
DATA PI/3.14159/
A=COS(EL)
AB=COS(BN)*COT(BN)/2.
BT=.1359-.7963*COS(EL)*SIN(EL)*COS(AZ)-.6462*(SIN(EL)**2)-.2313*((
QSIN(EL)*COS(AZ))**2)-.2549*(2.*(1.+COS(EL)-1.5*SEC(EL))-SEC(EL)*(
QSIN(EL)**2)*(1.+(COS(AZ))**2))
CT=.2549*(3.+2.*(SIN(EL)**2)*(1.+(COS(AZ))**2))
DT=-(.372*COS(EL)-.5*(1.-COS(EL)))
ET=A
BC=.1359+.7963*COS(EL)*SIN(EL)*COS(AZ)-.6462*((SIN(EL))**2)-.2313*
Q((SIN(EL)*COS(AZ))**2)+.2549*(2.*(1.+COS(EL)-1.5*SEC(EL))-SEC(EL)*
Q((SIN(EL))**2)*(1.+(COS(AZ))**2))
CC=CT
DC=-(.372*COS(EL)+.5*(1.-COS(EL)))
EC=ET
BH=.5*(1.+COS(EL)-1.5*SEC(EL)*(1.+(SIN(EL)*COS(AZ))**2))
CH=.75*(1.+(SIN(EL)*COS(AZ))**2)
DH=(A-1.)/2.
EH=A
WRITE(3,1000)A,AB,BT,CT,DT,ET,BC,CC,DC,EC,BH,CH,DH,EH
1000 FORMAT(2X,'SKY IRRADIANCE CONSTANTS'/2(2X,7(F10.4,2X)/))
RETURN
END
SUBROUTINE LUMIN(ES,BL,BR,BHS)
COMMON/FAD/SH,TC,PT,PG,SC,AT,RB
COMMON/CONST/A,AB,BT,CT,DT,ET,BC,CC,DC,EC,BH,CH,DH,EH
DATA PI/3.14159/
C ATTENUATION WITH SLIGHT HAZE, STANDARD ATMOSPHERIC HEIGHT
DATA CN,XN,ZT/.00015,1.6,8000./
C RADIANT FREQUENCY RESPONSE RANGE, VISUAL FUNCTION
DATA W0,DW/375.,5./
W=W0
ES=0.
BL=0.
BHS=0.1
C IRRADIANCE, MICROWATTS/CM SQ/5 NM
DO 10 I=1,79
W=W+DW
CALL STDBS(I,X,Y,Z)
CALL CRDS(I,RG)
CALL BKGD(I,RB)
CALL SKYI(I)
ES=ES+SH*Y*DW
BL=BL+SH*PB*Y*DW/PI
AK=CH*((530./W)**XN)
AW=1./(ZT*AK)
BHS=BHS+SH*Y*DW*(BH+CH*AW+(DH+EH*AW)*RG)/(2.*PI*(1.+A*RG))
10 CONTINUE
C ILLUMINANCE, FOOTCANDLES
ES=ES*.6317
C BRIGHTNESS, FOOT LAMBERTS (MILLILAMBERTS)
BHS=BHS*1.9849
BL=BL*1.9849
BR=BL*PI/ES
RETURN
END

```

```

SUBROUTINE RADT(RX,RXD)
C COMPUTES RADIANT ENERGY FROM TARGET AND BACKGROUND
COMMON/RAD/SH,TC,RT,PG,SC,AT,RB
COMMON/CONST/A,AB,BT,CT,DT,ET,BC,CC,DC,EC,BH,CH,DH,EH
COMMON/RADD/WT(79),WB(79),SS(79)
DATA PI/3.14159/
C ATTENUATION WITH SLIGHT HAZE, STANDARD ATMOPHERIC HEIGHT
DATA CH,XN,ZT/.00015,1.6,8000./
C RADIANT FREQUENCY, 10 NM INTERVALS, 380 NM TO 770 NM
DATA H0,DH/370.,10./
W=H0
DO 10 I=1,79,2
W=W+DW
CALL GFDS(I,PG)
CALL BKGD(I,RB)
CALL TGTS(I,RT)
CALL SKYI(I)
CALL ATTC(I,AT)
AK=CN*((530./W)**YN)
AW=1./(ZT*AK)
C IRRADIANCE
ST=SH*(PI*RG+(BT+AW*CT+RG*(DT+AW*ET))/(1.+RG*A))/(2.*PI)
SB=SH*(1.+RG*AB)
SN=SH*(BH+CH*AW+RG*(DH+AW*EH))/(2.*PI*(1.+RG*A))
SS(I)=SH*(PI*RG+(BC+AW*CC+RG*(DC+AW*EC))/(1.+RG*A))/(2.*PI)
C RADIANCES
WTT=ST*RT/PI
WBB=SB*RB/PI
C APPARANT RADIANCES
WT(I)=WTT*EXP(-AT*RX)+SN*(1.-EXP(-AT*RX))
WB(I)=WBB*EXP(-AT*RXB)+CN*(1.-EXP(-AT*RXB))
10 CONTINUE
RETURN
END
SUBROUTINE SKYI(I)
COMMON/FAD/SH,TC,RT,RG,SC,AT,RB
DIMENSION S(79)
C CIE STANDARD ILLUMINANT, D-65
C TOTAL SKY IRRADIANCE, HORIZONTAL PANEL, FROM DAY-SKY AND SUN
C RELATIVE POWER
C MW/CM2/NM AT 5 NM INTERVALS, FROM 380 NM TO 770 NM
DATA S/50.,52.3,54.6,68.7,82.8,87.1,91.5,92.5,93.4,90.1,86.7,95.8,
A104.9,110.9,117.,117.4,117.8,116.3,114.9,115.4,115.9,112.4,108.8,
R109.1,109.4,108.6,107.8,106.3,104.8,106.2,107.7,106.,104.4,104.2,
C104.,102.,100.,98.2,96.3,96.1,95.8,92.2,88.7,89.3,90.,89.8,89.6,
D88.6,87.7,85.5,83.3,83.5,83.7,81.9,80.,80.1,80.2,81.2,82.3,80.3,
E78.3,74.,69.7,70.7,71.6,73.,74.3,68.,61.6,65.7,69.9,72.5,75.1,
F69.3,63.6,55.,46.4,56.6,66.8/
C RELATIVE FACTOR, 1.7X
SH=1.7*S(I)
RETURN
END

```

```

SUBROUTINE GRDS(I, RG)
C SPECTRAL REFLECTANCE VALUES FOR GROUND COVER
  DIMENSION R(79)
C SOURCE PENNDORF 1956 VEGETATIVE TYPE C-1D FOREST AUTUMN
  DATA R/.047,.048,.049,.05,.051,.052,.053,.054,.056,.058,.06,
    A3*.061,.062,.063,.064,.067,.069,.062,.076,.078,.079,.081,.083,
    B.087,.091,.097,.112,.123,.134,.142,.151,.159,.168,.173,.178,.184,
    C.191,.192,.193,.195,3*.196,.193,.191,.192,3*.193,.192,.19,.191,
    D.193,.202,.212,.223,.234,.246,.259,.272,.285,.3,.315,.33,.345,
    E.361,.378,.393,.409,.426,.443,.452,.46,.47,.481,.491,.5/
  RG=R(I)
  RETURN
END
SUBROUTINE TGTS(I, RT)
  DIMENSION P(79)
C TARGET SPECTRAL REFLECTANCE VALUES, LUSTERLESS O.D. PAINT 34087
  DATA R/5*.0409,.0414,.0427,.0436,2*.0445,.045,.0454,.0464,.0473,
    1.0479,.0491,.0509,.0527,.0536,.0554,.0564,.06,.0618,.0636,.0645,
    2.0682,.0727,.0745,.0773,.08,.0809,.0818,4*.0827,4*.0836,2*.0827,
    3.0836,.0854,.0891,.0909,.0927,.0945,.0973,.0991,.1,.1009,.1018,
    4.1054,.1064,.1082,.1091,.11,.1145,.1173,.119,.1236,.1273,.1318,
    5.1364,.1436,.1473,.1527,.1554,.1636,.1673,.1736,.1818,.1909,.2,
    6.2091,.2182,.2273,.2364/
  RT=R(I)
  RETURN
END
SUBROUTINE BKGD(I, RB)
C SPECTRAL REFLECTANCE VALUES FOR TARGET BACKGROUND
  DIMENSION R(79)
C SOURCE KRINDV 1953 TYPE III FIELD GRASS AUTUMN
  DATA R/.03,.0305,.031,.0315,.032,.0325,.033,.0345,.036,.0375,.039,
    U.0405,.042,.0425,.043,.044,.045,.046,.047,.048,.049,.0495,.05,.051
    U,.052,.054,.056,.059,.062,.0665,.071,.074,.077,.0785,.08,.0805,
    U.081,.0805,.08,.079,.078,.077,.076,.0765,.077,.078,.079,.0795,.08,
    U.081,.082,.082,.082,.081,.08,.081,.082,.083,.084,.092,.099,.108,
    U.116,.128,.14,.154,.167,.178,.189,.203,.216,.238,.259,.267,.275,
    U.285,.295,.323,.35/
  RB=R(I)
  RETURN
END

```

```

SUBROUTINE ATTG(I,AT)
C ATMOSPHERIC HORIZONTAL ATTENUATION COEFFICIENT (1/KM)
  DIMENSION A(79)
C VISIBILITY 23.5 KM STANDARD CLEAR
  DATA A/.254,.2505,.247,.2435,.24,.2365,.233,.2295,.226,.2225,.219,
1,.2155,.212,.2085,.205,.203,.201,.199,.197,.195,.193,.191,.189,.187
2,.185,.183,.181,.179,.177,.175,.173,.171,.169,.167,.165,.163,.162
36,.1614,.1602,.159,.1578,.1566,.1554,.1542,.153,.15235,.1517,.1510
45,.1504,.14975,.1491,.14845,.1478,.14715,.1465,.14585,.1452,.14455
5,.14390,.14325,.1426,.14195,.1413,.14065,.14,.1395,.139,.1385,.138
6,.1375,.137,.1365,.136,.1355,.135,.1345,.134,.1335,.133/
  AT=A(I)/1000.
  RETURN
END
SUBROUTINE STDBS(I,XS,YS,ZS)
C STANDARD OBSERVER CIE 1931 ONE-DEGREE TGT
C 380NM TO 770NM IN 5NM INCREMENTS
  DIMENSION X(79),Y(79),Z(79)
  DATA X/.0014,.0022,.0042,.0076,.0143,.0232,.0435,.0776,.1344,.2148
1,.2839,.3285,.3483,.3481,.3362,.3187,.2908,.2511,.1954,.1421,.0956
2,.058,.032,.0147,.0049,.0024,.0093,.0291,.0633,.1096,.1655,.2257,
3,.2904,.3597,.4334,.5121,.5945,.6784,.7621,.8425,.9163,.9786,1.0263
4,1.0567,1.0622,1.0456,1.0026,.9384,.8544,.7514,.6424,.5419,.4479,
5,.3608,.2835,.2187,.1649,.1212,.0874,.0636,.0468,.0329,.0227,.0158,
6,.0114,.0081,.0058,.0041,.0029,.002,.0014,.0010,.0007,.0005,.0003,
7,.0002,.0002,.0001,.0001/
  DATA Y/.0,.0001,.0001,.0002,.0004,.0006,.0012,.0022,.0040,.0073,
1,.0116,.0168,.023,.0298,.038,.048,.060,.0739,.091,.1126,.139,.1693,
2,.208,.2586,.323,.4073,.503,.6082,.71,.7932,.862,.9149,.954,.9803,
3,.995,1.0002,.995,.9786,.952,.9154,.87,.8163,.757,.6949,.631,.5668,
4,.503,.4412,.381,.321,.265,.217,.175,.1382,.107,.0816,.061,.0446,
5,.032,.0232,.017,.0119,.0082,.0057,.0041,.0029,.0021,.0015,.001,
6,.0007,.0005,.0004,.0003,.0002,.0001,.0001,.0001,2*0./
  DATA Z/.0065,.0105,.0201,.0362,.0679,.1102,.2074,.3713,.6456,1.039
1,1.3856,1.623,1.7471,1.7826,1.7721,1.7441,1.6692,1.5281,1.2876,
2,1.0419,.813,.6162,.4652,.3533,.272,.2123,.1582,.1117,.0782,.0573,
3,.0422,.0298,.0203,.0134,.0087,.0057,.0039,.0027,.0021,.0018,.0017,
4,.0014,.0011,.001,.0008,.0006,.0003,.0002,.0002,.0001,2*0./
  XS=X(I)
  YS=Y(I)
  ZS=Z(I)
  RETURN
END

```

LIST OF NOTATIONS AND SYMBOLS

$r_0$	spectro-reflectance of the terrain ground cover
$r_t$	spectro-reflectance of the target surface
$r_b$	spectro-reflectance of background
$t_\lambda$	spectro-transmittance of the coating material
$s_\lambda$	spectro-scattering coefficient of the coating material
$x, y, z$	coordinates of a right-hand cartesian coordinate system located at the observer with the x-axis in the target direction and the z-axis vertically toward the sky
$z_t$	thickness of the atmosphere
$\zeta$	zenith angle measured from the z-axis
$\alpha$	azimuth angle measured from the x-axis in a clockwise direction about the positive z-axis
$\zeta_s$	zenith angle of the sun
$\alpha_s$	azimuth angle of the sun
$\lambda$	wavelength of monochromatic radiant energy
$I_{O\lambda}$	intensity of sunlight at the ozone layer
$N_\lambda$	sky radiance
$\beta_\lambda$	atmospheric extinction coefficient
$\sigma_{\phi\lambda}$	atmospheric volume scattering coefficient
$\phi$	scattering angle
$\eta$	forward scattering coefficient
$N_\lambda^p$	primary sky radiance
$N_\lambda^m$	multiple scattered sky radiance
$H_\lambda^s$	sky irradiance
$N_\lambda^h$	horizontal sky radiance

(Continued)

$W_{\lambda}^g$ , ground radiance  
 $W_{\lambda}^T$ , target radiance  
 $H_{\lambda}^T$ , target irradiance  
 $H_{\lambda}^{TS}$ , target irradiance from sky  
 $H_{\lambda}^{Tg}$ , target irradiance from ground  
 $\rho$ , ground radius  
 $\ell$ , straight line distance  
 $h_T$ , target height above ground level (x-y plane)  
 $\sigma_{\lambda}$ , ground level atmosphere extinction coefficient  
 $d\omega$ , differential element of angular area  
 $ds$ , differential element of ground area  
 $\theta$ , angle between z-axis and radiating point  
 $\psi$ , angle between surface normal and radiating point  
 $n$ , surface normal  
 $\gamma$ , angle between incline of background surface and z-axis  
 $W_{\lambda}^b$ , background radiance  
 $H_{\lambda}^b$ , background irradiance  
 $H_{\lambda}^{bs}$ , background irradiance from sky  
 $H_{\lambda}^{bg}$ , background irradiance from ground  
 $h_b$ , height of background on observer-target line above ground level (x-y plane)  
 $\psi_s$ , angle between surface normal and radiating sky point  
 $\psi_g$ , angle between surface normal and radiating ground point  
 $H_{\lambda}^c$ , coating irradiance  
 $H_{\lambda}^{cs}$ , coating irradiance from sky  
 $H_{\lambda}^{cg}$ , coating irradiance from ground

(Continued)

$h_c$ , height of coating above ground level (x-y plane)  
 $W_{\lambda}^T$ , apparent radiance at telescope from target through intervening atmosphere  
 $W_{\lambda}^b$ , apparent radiance at telescope from background through intervening atmosphere  
 $X_T$ , x-axis position of target  
 $X_b$ , x-axis position of interception point of background and observer-target line  
 $l_T$ , straight line distance between observer and target  
 $l_b$ , straight line distance between observer and background  
 $a_x$ , directional cosine along x-axis of observer-target line  
 $a_z$ , directional cosine along z-axis of observer-target line  
 $\omega'_T$ , apparent angular area subtended by target at observer's position  
 $A_T$ , area of target  
 $\mu$ , geographical latitude of observer in global coordinate system  
 $\delta$ , declination of the sun  
 $t$ , meridian angle  
 $t_a$ , local apparent time  
 $\alpha_N$ , bearing of observer-target line measured from true North

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